Temporal prediction of malaria in Apartadó through a probabilistic random walk

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Abstract

Introduction: the analysis of the behavior of malaria dynamics has helped to develop causal methods which have very accurately predicted the number of people infected with this disease annually.

Objective: to predict the monthly behavior of malaria in the city of Apartadó, Colombia, using the probabilistic random walk.

Method: the number of cases of malaria registered in Apartadó and reported monthly by SIVIGILA were obtained, and their analogy with the random walk was evaluated. Then, the non-equiprobability of the phenomenon obtaining monthly predictions using a second-class equation was determined.

Results: the analogy between the random walk and the monthly behavior of malaria in Apartadó was proven, and therefore the method was used to predict the number of cases in consecutive months in 2018, achieving a predictive accuracy ranging from 84.4-100%.

Conclusions: the behavior of malaria in the city of Apartadó has mathematical sequences that allow for its spatial-temporal prediction using the probabilistic random walk, allowing it to be used to guide epidemiological surveillance activities and observe the effectiveness of interventions on public health. (Acta Med Colomb 2023; 48. DOI: https://doi.org/10.36104/amc.2023.2538).

Keywords: probability, malaria, prediction.

Introduction

In epidemiology, models are commonly constructed linking biological and statistical aspects of infectious diseases to analyze their behavior, as has been done with models that estimate the basic reproduction number (R_0) to estimate disease dissemination. This type of information has been useful for identifying the burden of disease and establishing public health strategies to prevent or limit its occurrence (1, 2). However, one of the limitations of these models is that they are not generalizable because they are not universally reproducible.

These limitations are not foreign to the study of the malaria epidemic worldwide (3, 4). As pointed out by Li et al. (2011) (5), a single R_0 model for malaria can be established, generating multiple R_0 values depending on the method used, taking 1 as the reference threshold, which indicates the tendency for the disease to increase or decrease its incidence if its values exceed the threshold or are increasingly below the threshold, respectively. Furthermore, this study shows that when R_0 is applied, it should be accompanied by the respective considerations of the calculation method used and its assumptions because, otherwise, its calculation is meaningless (5).

Other malaria studies have been based on an analysis of climate variables like rain patterns, as seasonal variations in disease transmission have been described in relation to the rainy season. Similarly, temperatures between 20°C and 30°C have been positively related to transmission. Furthermore, it has been proven that other factors may be involved in the dynamics of the epidemic, such as changes in humidity, since mosquito longevity is greater when humidity is high (6).

In order to overcome the limitations of epidemiological methods like compartmental methods (3) and others based on climate analysis (6) or those that incorporate machine learning (7), methods have been developed that contribute to predicting the behavior of epidemics through *random walks*, based on the study of Brownian motion with apparently chaotic movements (8-10), and *probability theory*, with which the possibility of an event occurring can be measured (11-13).

Random walks (also known as probabilistic walks), as defined by Norbert Wiener, are a far-reaching method with applications in various disciplines, including mathematics, statistics, physics, economics and informatics. They are used to describe stochastic processes in which an object or entity experiences random movements within a defined area (14, 15).

In a random walk, an object, like a particle, financial entity or artificial intelligence agent, moves randomly in different directions. Each individual movement is determined accord-

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Correspondencia: Dra. Sandra Catalina Correa-Herrera. Bogotá D.C. (Colombia) E-Mail: grupoinsight2025@gmail.com Received: 29/XI/2024 Accepted: 22/IX/2023 ing to a specific probability distribution. There can be two types of walks: discrete, in which the movements are made in discrete steps (such as movements up or down in a network), or continuous, in which the movements occur in a continuous direction, such as a point that moves in a plane without restrictions (16, 17).

The usefulness of a random walk extends to various disciplines to describe random processes in order to understand, predict and optimize a wide range of phenomena and systems such as: molecular diffusion modeling (18), economics and finances (19), meteorological forecasts (20), social network and data analysis (21), image processing and computer vision (22), optimization and searching (23), biology and ecology (24), or cryptography, among others (25).

As a result of this theoretical focus, a random walk method has been developed which allowed the first temporal prediction of the number of people infected with malaria in Colombia in 2007, with a 95.6% success rate (26). Similar results were found in the temporal prediction of the number of people infected with dengue (27) and other events of interest, such as mortality secondary to car accident injuries (28).

The method's effectiveness in predicting the dynamics of the number of cases of malaria in Colombia at a national and annual level suggests its potential scale up to shorter periods of time and the municipal level. Therefore, the purpose of this study was to apply this method in predicting the number of cases per month for 2018 in the city of Apartadó, department of Antioquia, and compare the predicted values with the values reported by the Colombian National Epidemiological Surveillance System (SIVIGILA, in Spanish), to determine its success rate and validate its usefulness and possible applicability as a public health decision-making tool at a municipal level.

Materials and method

Type of study

A retrospective, observational study was carried out implementing a monthly time-series predictive method related to the number of people infected with malaria in the city of Apartadó, Antioquia, Colombia in 2018.

Population

The monthly figures of confirmed and notified cases of malaria between 2015 and 2019 from the city of Apartadó, Antioquia, were taken from the data compiled in SIVIGILA and the Sectional Health Department of Antioquia. Data from these years was chosen because it had been previously reviewed by the entities conducting epidemiological surveillance, allowing the malaria epidemic to be studied through the agents in charge of this subject.

Procedure

Initially, a geometric analogy was established between the random walk and the monthly behavior of the number of infected cases, according to the method previously established by Rodriguez et al. (2009, 2017, 2020) (26-28). Subsequently, the lengths of the monthly variations in the number of infected people were established using Equation 1:

$L = \sqrt{(X_0 - X_0)^2 + (Y_0 - Y_1)^2} Ecuación 1$

For the previous equation, X_0 and Y_0 refer to the coordinates of the initial month's value, and X_0 and Y_1 refer to those of the subsequent month. The X coordinates are the same in all months because movement is only studied in the Y axis, that is, the variation in infected people.

Consequently, a probability area was established in which the lengths found with Equation 1 were considered as probability events, calculated with Equation 2, which establishes a division between the length of the monthly number of people infected over the sum of all the lengths.

$$P(L) = \frac{Longitud \ cambio \ del \ número \ de \ infectados \ mensual}{Total \ longitudes} = \frac{L}{TL}$$

Ecuación 2

Then, the probability of the monthly number of infected people, P(N), was calculated using Equation 3, in which the monthly number of infected people in a given month was divided by the sum total of these changes.

$$P(N) = \frac{Valor del número mensual de infectados de malaria}{Total de casos reportados en el año} Ecuación 3$$

To determine the equiprobability of the dynamic of the values found, the mean square deviation was employed using Equation 4:

$$P(Rn) = \frac{N\acute{u}mero\ casos\ mensual}{Totalidad\ de\ casos\ } \pm \frac{1}{2\sqrt{N}}\ Ecuación\ 4$$

The behavior of the calculated mean square deviation values helped identify if, in this case, the malaria dynamics in Apartadó had values of infected people that were more likely to occur than others, allowing predictions to be obtained, as described in the literature (14-16).

Following this, a probability area was developed which considered the three values prior to the month to be predicted and with which a mathematical mean was calculated, that was plugged into Equation 1, which was simplified until Equation 5 was obtained in its quadratic form, which yielded two possible solutions:

Y_{Mes a predecir}

= -	$2Y_{Mes\ anterior} \pm $	$(-2Y_{Mes\ anterior})^2 - 4\{Y_{Mes\ anterior}^2 + (X_1 + X_2)\}$	$(-X_0)^2 - [P(L)^2 \times (TL)^2]$
		2	· · ·
			Ecuación 5

Where PL is the mathematical mean of the probability for the last three months and TL is the sum of the three lengths from the last three months.

Finally, in order to determine which of the two solutions yielded the most probable value, a probability area was created studying the occurrence of reductions or increases in the number of infected people compared to the previous month in bi-monthly and quarterly time periods. In addition, to confirm the method's predictive ability, the predicted value was compared with the values reported by SIVIGILA and its predictive accuracy was measured. This procedure was done to validate the method's ability to predict the behavior of the number of people infected throughout 2018, specifically the consecutive monthly behavior over six months (the second semester of the year).

Results

The monthly notifications of malaria cases in Apartadó in 2018 ranged from 20 to 87, while the calculated lengths ranged from 2 to 47, with corresponding length probabilities of 0.002-0.0636. The mean square deviations ranged from -0.0017–0.049 with a difference in expected values of -0.01–0.015 (Table 1), indicating that the behavior of the malaria epidemic in Apartadó is non-equiprobable.

Subsequently, the predictive ability of the established method was determined by predicting six consecutive months during 2018. Thus, the values of the number of infected people between July and December were predicted. After doing the calculations shown in Table 1, Equation 5 was applied, obtaining the ranges within which the value to be predicted lies, which corresponded to 11-83, 24-88, 18-62, 33-51, 37-51 and 37-43, respectively.

Table 1. Length, probability and mean deviation values in Apartadó during 2018.

After analyzing the increases and decreases for the consecutive months described, which corresponded to: increase, decrease, increase, increase, decrease and increase, respectively, a mathematical mean was calculated between the average obtained, the range and its upper limit (if there was an increase from the previous probability area), or the average and the lower limit (when there was a decrease). In this way, predictive values of 65, 40, 42, 44, 40 and 49 were obtained, which, when compared with the values reported for the same months (56, 40, 51, 47, 47 and 41) showed a predictive accuracy of 86.4%, 100, 82.6, 94.6, 84.50 and 84.4%, respectively, which indicated an average predictive accuracy of 89% (Table 2).

Discussion

This is the first published study of local data using a method to predict the monthly dynamic of malaria in the city of Apartadó, Antioquia, Colombia, in 2018 based on a random walk and using time-series constructed with historical SIVIGILA records. This methodology had 89% success in consecutive predictions over six months (the second half of 2018). It is important to note that these predictions were able to be made because it was first confirmed that, at the city level,

Month	People infected	L	P(L)	P(N)	MSD+	MSD-	MSD+ P(N)	MSD- P
January	41	12	0.016	0.018	0.029	0.008	0.011	-0.011
February	34	7	0.009	0.015	0.026	0.005	0.011	-0.011
March	20	14	0.019	0.009	0.019	-0.002	0.011	-0.011
April	40	20	0.027	0.018	0.028	0.007	0.011	-0.011
May	87	47	0.064	0.039	0.049	0.028	0.011	-0.011
June	47	40	0.054	0.021	0.031	0.010	0.011	-0.011
July	56	9	0.012	0.025	0.035	0.014	0.011	-0.011
August	40	16	0.022	0.018	0.028	0.007	0.011	-0.011
September	42	2	0.003	0.019	0.029	0.008	0.011	-0.011
October	44	2	0.003	0.020	0.030	0.009	0.011	-0.011
November	40	4	0.005	0.018	0.028	0.007	0.011	-0.011
December	49	9	0.012	0.022	0.032	0.011	0.011	-0.011
L: length; P: probability; MSD: mean square deviation.								

Table 2. Predicted percentages for six consecutive months in 2018.

	Months in 2018								
	July	August	September	October	November	December			
Reported	56	40	42	44	40	49			
Predicted	65	40	51	47	47	41			
Percentage accuracy	86.40%	100%	82.60%	94.60%	84.50%	84.40%			

the malaria epidemic had mathematical behavior characteristics that could be analyzed through a random walk.

Apartadó is a city in the Urabá region of Antioquia, which has recently been affected by endemic malaria. According to data published by the Epidemiological Bulletin of the National Institute of Health, in epidemiological week 43 of 2022 (October 23 to 29), Apartadó was one of two cities with a malaria outbreak in the country, with 30 notified cases expected and 62 notified cases found (29). Likewise, according to reports from the Sectional Health Directorate of Antioquia, in 2018, Apartadó was one of the cities most affected by malaria in the department of Antioquia, along with El Bagre, Vigía del Fuerte, Remedios, Zaragoza and 16 other cities at high risk of malaria (30).

The results of this study indicate that the disease dynamics in the city have a mathematical and physical order, which means that they can be approached using methods with a noncausal predictive perspective, with potential usefulness for public health surveillance authorities in guiding toward effective interventions to reduce the disease burden and transmission dynamics. In this regard, the public health usefulness of this methodology has been shown to arise from its exploration of the spatial temporal growth of an epidemic. A simple random walk system has proven to generate a notinsignificant dynamic compared to traditional epidemiological models (31). This method can also be useful for the numerical calculation of phase diagrams that describe the long-term behavior of epidemics. Likewise, the functional dependence of the basic reproduction number R₀ on the parameters that define this type of model shows the role of spatial fluctuations and, if the information is complemented with a random walk, could lead to a novel application or expression for $R_0(31)$, which could be useful for municipal surveillance authorities' decision making. Furthermore, if needed, attention can be given to future random simulations related to interregional transmission phenomena.

The results of this study were shared with local academic tropical medicine authorities and municipal public health surveillance decision-making staff. Discussing the method and results has raised great interest in complementing the analyses done using other methods such as the endemic index, timeseries, and construction of endemic channels, among others.

Before, this method had been used to predict the dynamics of malaria and dengue epidemics at the national level in 2007, reaching a predictive accuracy of more than 90% in both cases (26, 27). These studies implemented an analysis of annual dynamics; that is, the cases reported by the epidemiological surveillance authorities in the whole country for an entire year. In contrast, our study makes a predictive estimate maintaining this high accuracy on a smaller spatial and time scale, with consecutive monthly predictions for a city in Colombia. Thus, the current study shows that the method can be applied in smaller contexts and that it could potentially be aimed at the epidemiological surveillance authorities' needs at a municipal level.

Many complex approaches have been developed to study the behavior of infectious diseases, including malaria, which seek to attribute causal relationships (like changes in temperature or sociocultural behaviors, among other factors) to the rise or fall in case numbers in a country or region. For instance, machine learning-based studies have been done to establish climate change analyses and thus predict if the cases in one region of a country will increase or decrease (32, 33) or, on an even smaller spatial scale, cases in populated urban centers (34). These methods have proven to be highly accurate and make up for some of the deficiencies in compartmental models (1). However, they may be limited by the amount of data required to make a prediction (1) or the prediction may even be restricted to the specific region where the method has been applied, and therefore a globally applicable method cannot be assembled.

Our study, on the other hand, shows that analyses can be established to simplify the behavior of different diseases, which may be endemic in different areas, as is the case of malaria in Colombia. Likewise, these analyses can be conducted at a national and municipal level. Also, as this method is standardized, it is objective and reproducible and, therefore, specialized software or extensive retrospective or historical data are not required. However, we emphasize the importance of carrying out more studies to confirm the effectiveness and accuracy of the method for predicting the behavior of epidemics in different time and space scales.

The type of study we carried out follows the thinking of current theoretical physics and mathematics, which has allowed predictive methods to be developed in other fields of biomedical knowledge such as infectious disease, in which a method has been designed to predict CD4+ lymphocyte counts (35). Similarly, diagnostic methods have been developed to link fractal and Euclidean geometry in the context of arterial restenosis and cervical cancer (36, 37). Furthermore, using fractal geometry and dynamic systems, neonatal and adult cardiac dynamics have been diagnosed (38, 39). These studies show that theoretical approaches are useful for dealing with biomedical phenomena through objective and reproducible mathematical methods.

The limitations to be considered include the possibility of underreporting in SIVIGILA, which could result in small variations in the final predictive values. However, since a general dynamics pattern has been identified, it is possible that, in the future, the predictions themselves may help detect potential underreporting. This could be feasible, since the method is based on physical and mathematical principles, which means that the results do not depend on multifactorial models. Rather, by adopting a non-causal approach, the analysis of the phenomenon is simplified (40), and a probabilistic non-causal dynamic is used to make predictions about the number of people affected by malaria in the city.

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